Hierarchies and Economic Growth

Benjamin Bental*

Zvi Winer†

February 6, 2013

*Department of Economics, University of Haifa. bbental@econ.haifa.ac.il
†Department of Economics, Western Galillee College. ZvikaV@wgalil.ac.il

Abstract

This paper studies the interrelationship between the size of firms, their hierarchical structure and economic growth. The hierarchical structure is a consequence of the hidden nature of effort. Production workers need to be supervised to prevent shirking, but supervisors need also to be supervised for the same reason. Accordingly organizations trade-off the number of production workers and their effort against the overhead associated with the hierarchical structure needed to control the system. This tradeoff is affected by the amount of capital firms employ. Since capital and effective labor are complements, increased capital intensity implies larger firms with longer hierarchical structures. Embedding this setup in a simple growth model, we show that along the growth path firms are growing, their number is declining, productivity is rising, hierachies are deepening and wage gaps are increaisng. These features match some key stylized facts.

JEL E23,L23,O40

KEYWORDS: Hierarchies, economic growth.

Introduction

The hierarchical structure of organizations has been analyzed mainly from two points of view. One strand of the literature views the organization as a network which controls and analyzes information flows, whereby the optimal structure is designed to best process these flows.[1] The other strand thinks of hierarchies as a way to solve internal incentive problems.[2] This paper embeds a simple version of the basic models in the latter literature in a neoclassical growth environment. Its results are shown to be consistent with some key stylized facts pertaining to the evolution of the number of firms, their size and hierarchical structure.

Specifically, the current paper is based on work by Williamson (1967), Calvo and Wellisz (1979) and Qian (1994). These models seek to determine the optimal hierarchical structure of organizations focusing solely on internal costs rather than market constraints.[3] In Williamson's model production workers need to be "controlled" by their supervisors. Supervisors too need to be controlled by their supervisors, etc. Since every supervisor can control an exogenously given fixed number of subordinates, increasing the number of production workers requires more supervisors, supervisors of supervisors etc., leading to increased organizational costs. The latter are affected by two important factors. First, wages are assumed to grow at an exogenously given geometric factor along the hierarchical ladder. Second, cumulative control over production workers gets diluted down the ladder. Consequently, Williamson shows that a profit-maximizing organization, even if it faces a perfectly elastic demand, has a finite size and number of hierarchies.

Calvo and Wellisz (1979) have endogenized the control span of supervisors and the wage structure. However, they do not solve for the optimal size of the organization (which, absent

[1] An early contribution to this literature is Keren and Levhari (1979). Later contributions include, among others, Radner (1993) and Bolton and Dewatripont (1994) and the recent papers by Caliendo and Rossi-Hansberg (2011).

[2] See Mookherjee (2012) for a summary of this literature.

[3] In contrast, organizations in the Caliendo and Rossi-Hansberg (2012) setup face downwards sloping demand curves. Therefore their size is finite despite the fact that the lower envelope of average production costs (resulting from information processing) is decreasing with size.

demand constraints, turns out to be infinite. See Mookherjee 2012). In their analysis Calvo and Wellisz pose an incentive problem. Any worker's effort (at all levels) can be ascertained only if he or she is directly observed by a supervisor. Since effort is associated with some cost, workers face a choice between two options. They may deliver the expected level of effort, thereby assuring their pay, or shirk and risk being detected and fired. Given the workers' incentive constraint, the organization trades-off higher wages against tighter supervision. The latter depends on the control span of the supervisors and their supervisory "ability". In the final analysis, Calvo and Wellisz find the optimal ability and number of employees at each hierarchical level, showing that ability and wages are rising along the hierarchical ladder.

While both Williamson and Clavo and Wellisz allow the organization to choose the number of production workers, in Qian's (1994) paper that number is given. Like Calvo and Wellisz, Qian assumes that effort is a hidden action which becomes public information only if a worker is inspected. However, unlike Calvo and Wellisz, in Qian's framework detection probability depends solely on the control span of the supervisor. Similar to Williamson's formulation, output depends on the multiplicative effect of effort along the entire hierarchical ladder. Given the number of production workers this setup enables Qian to find the optimal number of hierarchies as well as effort and associated wage at each level. Qian's setup also results in increasing effort and wages along the hierarchical ladder.

Similar to Caliendo and Ross-Hansberg (2102), our model endogenizes the number both of hierarchies and workers at all levels, including production, in a general equilibrium setting. Caliendo and rossi-Hansberg model hierarchies as "problem solvers" of increasing expertise and study how changes in market structure (affected by international trade) impact, among other things, organizational structure. Our focus on the incentive rationale of organizational structures allows us to naturally embed these structures in a simple dynamic environment.

Specifically, we assume that production requires both capital and workers' effort, where the latter is complementary to capital. As a result, organizations employing more capital will be interesrted in inducing more effort. To achieve this goal, organizations increase supervision and add hierarchical levels. We show these effects by using a version of the Calvo and Wellisz inspection technology and Qian's labor contract. At the aggregate level capital is accumulated according to a simple growth process. Combining both model elements, we generate time-series of output, productivity, firm size and hierarchical structures which qualitatively mimic the data.

We use the dynamic environment of Bental and Demougin (2006) of an economy populated by infinitely-lived households. Each period households decide whether to be self-employed, or work for organizations. Households differ in their productivity as self-employed but are ex-ante identical as employees. Accordingly, households decide to become employees if they expect their market income to exceed their output as self-employed. Organizations are formed each period subject to the aforementioned production and supervision technology. They choose their hierarchical structure and offer level-specific contracts specifying wages, required effort and inspection probabilities. Taking incentive constraints into account, these contracts pay the minimal wage needed to induce the required effort. In addition to labor, organizations hire capital at the market interest rate. Finally, households allocate income generated from all sources between consumption and saving, which becomes next-period's capital in a standard way.

At any period, both the labor market and the capital market clear. These conditions determine the interest rate, which in turn simultaneously generate organizational structures and demand for capital. Along the growth path the economy accumulates capital and the interest rate decreases. Organizations employ more capital and labor and induce higher effort at the production level. Supervising more production workers who are required to exert higher effort necessitate more supervisors and higher levels of supervisory effort at all levels. At some point the addition of another hierarchical level becomes worthwhile.

At our specification these mechanisms cause expected wages offered by organizations to increase along the growth path, and the fraction of "employed" workers to grow. On the other hand, organizations become larger and their number declines. Furthermore, hierarchies are added, control spans grow, workers at all levels increase effort (thereby increasing productivity) and wage gaps along the hierarchical ladder become wider. These effects are driven solely by capital accumulation and abstract from technological change.

We argue that our results are consistent with several basic stylized facts. We provide some micro-evidence on the typical number of hierarchies and control spans found in the empirical micro-literature and show that we can quantitatively match these observations. In addition we provide some evidence, based on historical industry surveys in the U.S., which seems to indicate that organizations have been growing while their number (relative to the labor force) has been decreasing, at least for most of the 20th century. Increased labor productivity and widening wage gaps are, of course, well established facts.

The aforementioned stylized facts at both the micro and macro levels are presented in the following section. Section 3 presents the model In section 4 we discuss the equilibrium both statically and dynamically. In section 5 we present a parameterized version of the model and section 6 presents the model's numerical performance. We briefly conclude in section 7.

Stylized Facts

Micro-evidence

Studies providing direct evidence on the hierarchical structure of organizations are scarce. Delmastro (2002) researches 438 Italian manufacturing firms and finds 2-6 hierarchical levels, with an average of about 3. The econometric results imply that the number of corporate levels and control spans both increase with plant size.[4] Colombo and Delmastro (2008) report that in that sample average control spans range roughly between 8 and 12

[4] Based on an average control span of 7, Delmastro assesses that the growth of Siemens from 3000 to 57000 workers between 1890 and 1913 was associated with increasing the number of its hierarchical levels from 4 to nearly 6.

(Table 1.5). Rajan and Wulf (2006) study the top management structure of over 300 large publicly traded U.S. corporations from various industries and find that the average "depth" of the organizations, measured by the number of managers between the CEO and the lowest managers with profit center responsibility, has decreased from 1.58 in 1986 to 1.15 in 1999.[5] The control span of the CEO has increased during this period from an average of 4.2 to 7.9. They also find that CEOs whose span is above 10 earn more than twice the earnings of CEOs who directly control 4 subordinates or less. Façanha and Resende (2007) consider a sample of 6567 manufacturing firms in Brazil. They find that the number of hierarchical levels ranges between 1 and 10, with an average of 3.2. Finally, in their recent empirical study, Caliendo, Monte and Rossi-Hansberg (2012) identify up to four hierarchical levels in French firms.

While our model focuses on time dynamics rather than cross-section variability, we show below that it is qualitatively and quantitatively consistent with these characteristics: the number of hierarchical levels ranges between 1 and 5 and pay is increasing at increasing rates along these levels. Moreover, control spans range between 6 and 12, and are increasing with firm size.

Macro-evidence

Aggregate historical characteristics at the firm and plant levels are available from historical statistics of the United States, albeit only for the manufacturing sector. We use these data as indicators of developments that may have taken place in the business sector as a whole and illustrate some key trends which our model captures.

To indicate the extent to which manufacturing may be representative we present in Figure 1 the share of manufacturing value added in U.S. GDP and employment in the labor force.[6] The Figure indicates that both have been increasing until the mid 1960s, when the

[5] Rajan and Zingales interpret the last fact as evidence for a "flattening" trend in the structure of firms.

[6] Sources for figures 1-3: Historical Statistics of the U.S, Colonial Times to1970, Statiatics for Industry Groups and Industries: Annual Survey of Manufactures (2005), Economic Report of the President (2011).

trend has reversed.

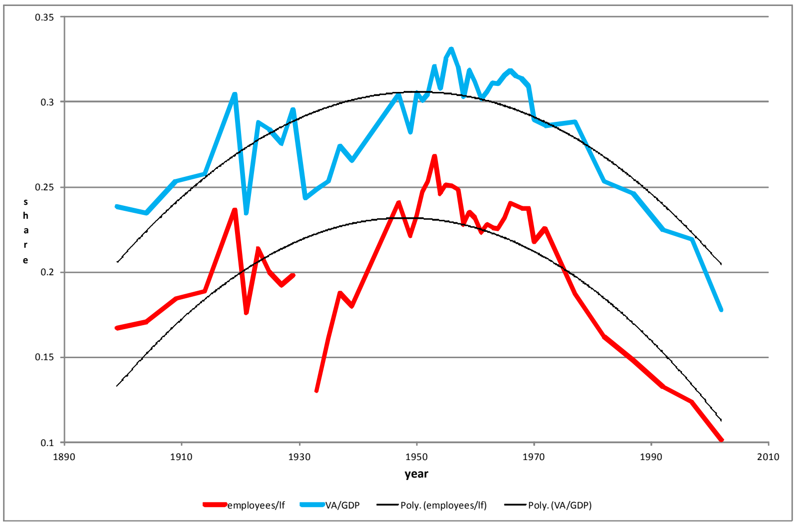

Figure 1: Manufacturing Share, Output and Employment

Figure 2 indicates that also establishments were growing until the mid 1960. The later trend reversal indicates that the decline of manufacturing's share in output had organizational repercussions, resulting in smaller establishments.

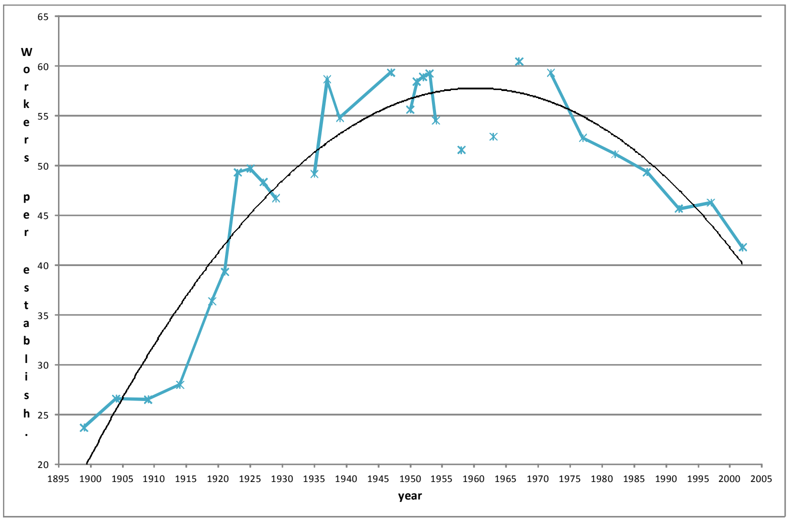

Figure 2: Workers per Establishment

Figure 3 depicts the familiar labor productivity (value added per worker) growth, and shows that despite the structural change during the last decades of the 20th century, labor productivity in manufacturing continued to increase.

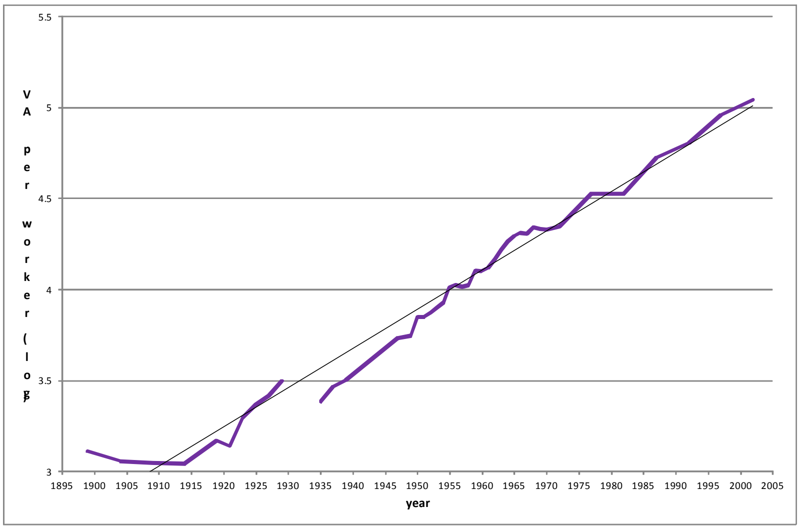

Figure 3: VA per Worker - Manufacturing (log)

Starting in 1977 there is some analogous evidence related to the entire business sector.[7] These data are depicted in Figure 4 and indicate that despite the decline in the manufacturing sector, in the business sector as a whole the average number of workers per firm has been increasing over the past three decades.

[7] US Small Business Administration (2010): Establishment and Employment by Employment Size of Firm, 1977-2009.

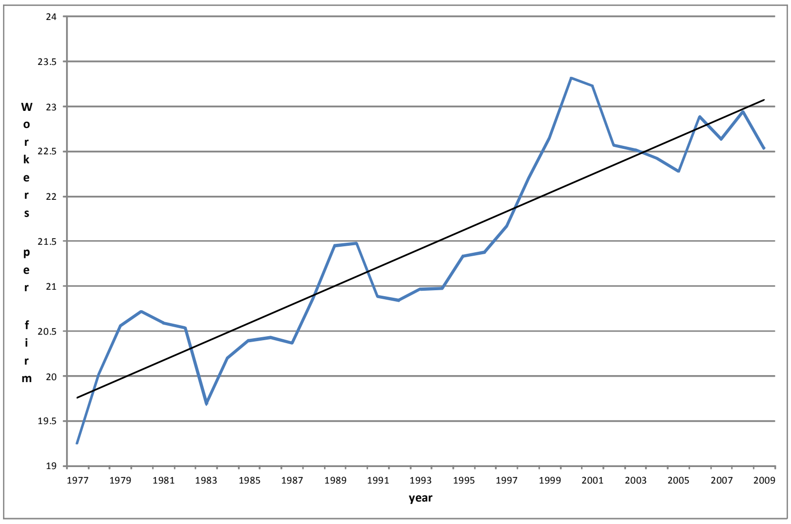

Figure 4: Workers per Firm, Business Sector

Finally we present some evidence concerning the number of manufacturing establishments relative to the entire labor force (in thousands).[8] Figure 5 is based on the historical data for the manufacturing sector, and while it is characterized by considerable fluctuations over time, especially during the Great Depression, the trend is clearly downwards.

Figure 5: Establishments/LF, Manufacturing

The decline of the number of manufacturing establishments during the last decades of the century may be associated with the aforementioned smaller share of that sector in the

[8]Labor force data are taken from the Bureau of Labor Statistics.

economy over-all. Therefore we provide also the more recent data on the number of firms taken from the US Small Business Administration. In Figure 6 we normalize this number by the participants in the labor force rather than the entire labor force.[9] Clearly, these data display considerable business-cycle fluctuations, but the trend seems to be downwards.[10]

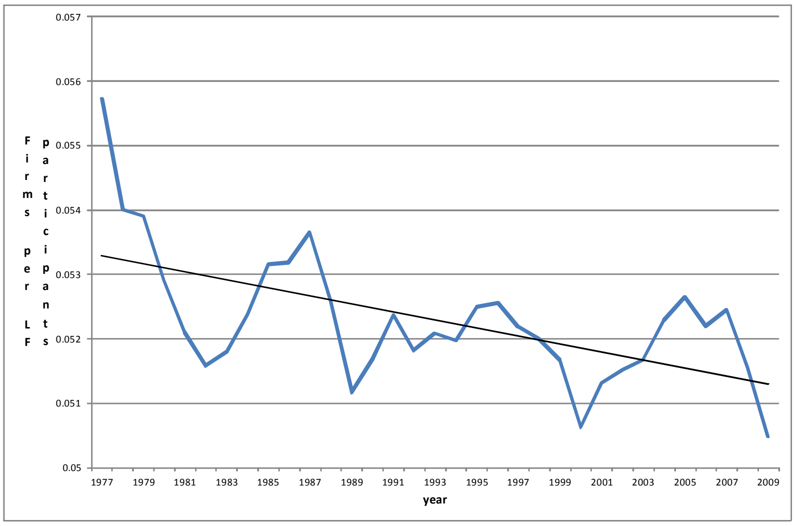

Figure 6: Firms/LF Participants

The model below completely abstracts from changes of the sectorial composition of output. It also ignores technological changes in production or management methods. Nevertheless the model is consistent with the main trends displayed in the data. It generates firms which become larger over time at a rate that outpaces their increased share in employment, so that their number is decreasing. In addition, the endogenous organizational changes discussed above induce higher labor productivity.

The model: households

We consider a single good, discrete-time economy with a constant population of infinitely lived households indexed on the unit interval by j. Each household consists of a continuum

[9] This is the normalization which is relevant for the model below, in which there is no distinction between the labor force and participants. Unfortunately data limitations prevented us from carrying out a similar normalization for the historical evidence.

[10] To gain further insight, notice that the share of firms employing up to 9 workers has declined from 78.9% in 1977 to 75.7% in 2009.

of identical members over the unit interval.[11] Each member of the household is endowed with 1 unit of labor per period. At period t every member i of the household may exert effort ε_t^i which affects that member's labor productivity in a way specified below. At the beginning of period t household j owns $s_t(j)$ units of capital which are inelastically supplied to the capital market at the rental rate r_t. In addition, households are endowed with claims on profits generated by firms in the economy.[12]

All households have identical intertemporal preferences, given by:

$$\sum_t \eta^t u(c_t),\ 0 < \eta < 1, \tag{1}$$

where

$$u(c) = \begin{cases} c\ if\ c \geq (1-\sigma)I,\ 0 < \sigma < 1 \\ 0\ otherwise, \end{cases} \tag{2}$$

and where I denotes income.[13]

Each period a household can either belong to the "self-employed" sector or decide to send its members to the "organized sector" (described below) as employees. While as employees all members of all households are ex-ante identical, as self-employed households differ in their members' skills. Specifically, every member of household j produces $y(j)$ units of the good, at no effort cost. Without loss of generality we order households according to their productivity as self-employed from the lowest to the highest. As employees members of the households are required to exert effort. We denote the income-equivalent utility cost associated with effort ε by a convex function $g(\varepsilon)$, with $g(0) = 0$.

[11] The basic structure is identical to that used by Bental and Demougin (2006). The household pools all risks associated with its members and thus faces no uncertainty.

[12] It is, of course, possible to introduce trade in these claims and price them. We ignore this issue because it has no bearing on our results.

[13] This specification clearly simplifies saving decisions and allows us to focus on the structural dynamics of the system.

Employment

If the household chooses the employment status, it sends its members to the labor market to be randomly matched with one of $H-1$ employment contracts.[14] The probability that a member will be matched with contract h is denoted by χ_h. Every contract h consists of a triplet $\{\varepsilon_h, w_h, p_h\}$. Here ε_h denotes the amount of effort an employee is required to exert, w_h is the wage paid unless the employee is caught shirking and p_h is the probability that the employee is monitored (see below).

We assume that every household member cares about his contribution to the household's income net of the associated effort cost. Based on these preferences, once matched with a contract, the household member autonomously rejects or accepts the contract, and in the latter case decides on the level of effort to produce. If w_h is positive, the contract is accepted, and the member will choose to produce either effort 0 or ε_h, so as to satisfy:

$$\max\left\{(1-p_h)w_h, w_h - g(\varepsilon_h)\right\}, \tag{3}$$

where the elements represent expected income net of effort cost if shirking or not, respectively. Consequently, the expected contribution of household members who are matched with contract h to the household's income, w^h, is:

$$w^h = \begin{cases} (1-p_h)w_h \; if \; \varepsilon = 0 \\ w_h \; if \; \varepsilon = \varepsilon_h, \end{cases} \tag{4}$$

and the total expected income of a household whose members are employees is:

$$y^E = \sum_{i=1}^{H-1} \chi_i w^i. \tag{5}$$

Consequently, household j decides to send its members to the labor market if and only if

[14] For clarity, time indices are omitted.

$y^E > y(j)$, and its income is given by:

$$\overline{y}(j) = \max\left\{y^E, y(j)\right\} \tag{6}$$

Saving

At any period t, once all household members have delivered their income, household j decides on the allocation of its total income between saving, $s_{t+1}(j)$, and consumption, $c_t(j)$, facing to the following budget constraint:

$$c_t(j) + s_{t+1}(j) \leq \overline{y}_t(j) + (1 + r_t)s_t(j) + d_t(j) \equiv I_t(j) \tag{7}$$

where $d_t(j)$ denotes all dividends accruing to the household.

Due to the intertemporal utility specification in (1) and (2), the saving rule is given by:

$$s_{t+1}(j) = \begin{cases} \sigma I_t(j)\ if\ 1 + r_{t+1} > \frac{1}{\eta} \\ 0\ if\ 1 + r_{t+1} < \frac{1}{\eta} \end{cases} \tag{8}$$

The model: firms

Firms are formed for a single period. They consists of $H \geq 2$ hierarchical levels. At the top (level H) there is a "management", which monitors hierarchy level $H-1$. Employees at that level monitor level $H-2$, etc. Production is carried out at the bottom, level 1. Output, Q, is given by a standard production function requiring as inputs labor, N_1, and capital, K, where labor's productivity depends multiplicatively on the workers' effort, ε_1:

$$Q = F(K, \varepsilon_1 N_1). \tag{9}$$

We assume that the production function is increasing and concave in both arguments, and that all inputs are essential, so that $F(0, \cdot) = F(\cdot, 0) = 0$.

Employees at all levels (except management) are required to exert effort which is (stochastically) monitored by their supervisors to avoid shirking. Specifically, production workers must be monitored by their supervisors, otherwise they shirk and output will be zero. These supervisors in turn must be monitored, otherwise they will not monitor the production workers who will not produce, etc.

Contracts, monitoring and profit maximization

As stated above, it is assumed that a firm can avoid paying a promised wage only if it is proven that the worker has not met the required effort level.[15] However, shirking can be observed and proven only if the worker is monitored. The workers take this fact into account and, given the terms of the contract, decide whether to shirk according to (3). Therefore, given a monitoring probability, p, the cost-minimizing wage that induces any specified effort level ε satisfies:

$$w = \frac{g(\varepsilon)}{p}. \tag{10}$$

In other words, when inducing any given level of effort, the firm faces a trade-off between paying higher wages and increasing (costly) monitoring.

Similar to Calvo and Wellisz (1979) we assume that the probability of being monitored, p_h, does not depend only on the inverse of the supervisor's control span. Here it depends also on their effort. Thus, we specify:

$$p_h = P_h(\varepsilon_{h+1} m_{h+1}),\ h = 1, ..., H-1, \tag{11}$$

where the inverse of the control span, m_{h+1}, is given by $\frac{N_{h+1}}{N_h}$ and N_h denotes the number of workers at level h.

Assumption 1: At the top level, $\varepsilon_H N_H = \gamma$ which is exogenously given and is independent of H.

[15] for a discussion of this assumption, see footnote 8 in Qian (1994).

The parameter γ reflects institutional features associated with the characteristics of corporate governance affecting the ability of top-management to control the organization. Here we take these characteristics as given.

Assumption 2: The functions $P_h(\cdot)$ have the following properties:

1. $P_h(0) = 0$

2. $P(\cdot)$ is increasing and concave.

3. $\lim_{\varepsilon m \to \infty} P_h(\cdot) = 1$.

4. $P_{h+1}(\cdot) \leq P_h(\cdot)$, $h = 1, ..., H - 1$.

The first three properties reflect the idea that smaller control spans and higher supervisory effort increase monitoring probabilities. The last property captures the notion that supervision is likely to become harder at higher levels of the hierarchy.[16]

Altogether, taking into account the characteristics of the wage contract given by (10), in this environment the firm chooses the amount of capital, K, the number of hierarchies, H, and for $h = 1, .., H - 1$; employment levels, N_h, which also determine control spans, and effort levels, ε_h, so as to maximize profits, given by:

$$\pi = F(K, \varepsilon_1 N_1) - \sum_{h=1}^{H-1} N_h \frac{g(\varepsilon_h)}{P_h(\frac{\varepsilon_{h+1} N_{h+1}}{N_h})} - (r + \delta)K, \tag{12}$$

where δ is the rate of capital depreciation. In particular, these choices determine the employment contracts $\{\varepsilon_h, w_h, p_h\}$ offered to workers at the different hierarchies.

[16] In Garicano (2000) an analogous property emerges endogenously. There the rationale for the creation of hierarchical structure is to prevent knowledgable managers from dealing with simple questions that can be handled by less knowledgable people at lower levels.

Equilibrium

We divide the equilibrium into two stages, a *temporary equilibrium*, and then the *dynamic equilibrium*.

Temporary equilibrium

An informal discussion

In a temporary equilibrium we take the total amount of capital owned by households at the beginning of any period, $\overline{K}$, as a given. In equilibrium, this amount is employed by an endogenous measure ϕ of firms. The amount of capital employed by each firm completely determines its structure: the number of hierarchies, employment and effort levels. These determine, in turn, employment contracts. In addition, the likelihood that a random worker will face a particular contract is given by the fraction of employees working at each level of the hierarchy. Thus, households can compute the income they obtain by sending their members to the organized sector and compare it to the income they generate as self-employed. Since the former is independent of household characteristics and the latter is, there is a cutoff point which determines the fraction of households working for the organized sector, z.

According to the above, in the final analysis there are two market-clearing conditions that have to be met. First, firm demand for capital must equal the supply. The demand of any specific firm is determined by the interest rate. Thus, denoting the demand by $K(\cdot)$, we obtain that for any interest rate ρ, the number of firms that clears the capital market, $\phi^K(\rho)$, is given by:

$$\phi^K(\rho) = \frac{\overline{K}}{K(\rho)}. \tag{13}$$

The interest rate determines also the rest of the firm's structure, specifically the number of hierarchies and workers per hierarchy. Thus demand for labor of any given firm is $N(\rho) = \sum_{h=1}^{H(\rho)-1} N_h(\rho)$. On the other hand, the interest-rate dependent contracts and their distribution determine the fraction of households which choose to become employees, $z(\rho)$.

This creates another market clearing condition, namely:

$$\phi^N(\rho) = \frac{z(\rho)}{N(\rho)}. \tag{14}$$

The equilibrium interest rate, r, must be such that both the capital market and the labor market clear:

$$\phi^K(r) = \phi^N(r). \tag{15}$$

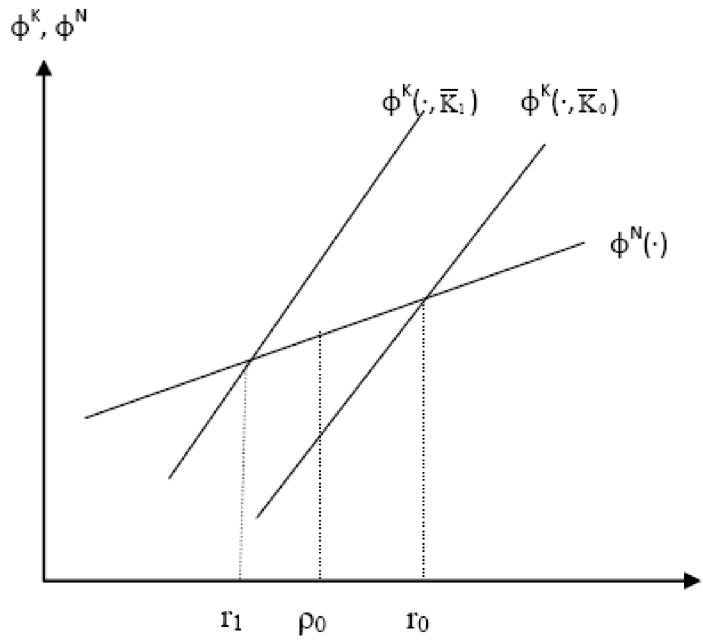

Figure 7: Temporary equilibrium

To provide intuition, assume that at some interest rate ρ_0 we have $\phi^K(\rho_0) < \phi^N(\rho_0)$. Suppose that the measure of firms $\phi^N(\rho_0)$ is used in (13). Clearly, this would imply that there is an excess demand for capital. It is reasonable to expect that the interest rate should increase, reducing the amount of capital any individual firm demands. By (13), ϕ^K increases. At the intensive margin, the lower amount of capital employed reduces also the demand for labor and wages at every hierarchy level (and at the extensive margin, may reduce the number of hierarchies), thereby lowering both z and N. Thus it is not clear whether z/N decreases. In any event, as long as the slope of $\phi^N(\cdot)$ is smaller than that of $\phi^K(\cdot)$, the gap between the two decreases when ρ increases. Furthermore, if $N(\cdot)$ decreases faster than $z(\cdot)$ we get the configuation shown in Figure 7. Under such circumstances, higher levels of $\overline{K}$, e.g. $\overline{K}_1 > \overline{K}_0$, generated, say, by a growth process, imply a decline in both the measure of firms and the interest rate ($r_1 < r_0$). This corresponds to the results obtained in the numerical experiment below.

A formal definition

Omitting the time index, a *temporary equilibrium* is defined as follows:

Definition 1. : Let z denote the fraction of households choosing to become employees and ϕ the measure of firms. A *temporary equilibrium* is a tuple $\mathbb{E}^T = \{\overline{y}(j), K, H, N_h, \varepsilon_h, w_h, p_h \chi_h,$

$z, \phi, r\}$, $j \in [0,1]$, $h = 1, ..., H-1$, such that, given the total amount of capital owned by households, $\int_0^1 s(j)dj$:

1. **Households**: Given $\{\varepsilon_h, w_h, p_h, \chi_h\}$, $h = 1, ..., H-1$, households chose their employment sector complying with their members' effort choices given by (3), and generate the corresponding income as given by (6).

2. **Firms**: Given r, firms maximize (12) which, together with (10), determines

   $K, H, N_h, \varepsilon_h, w_h, p_h,\ h = 1, ..., H-1.$

3. **Aggregate level**:

   i. The probability of a worker being matched with contract h is given by

   – $\chi_i = \frac{N_i}{\sum_{h=1}^{H-1} N_h}$, $i = 1, ..., H-1$.

   ii. The fraction of households choosing the employment status, z, solves:

   – $y(z) = \sum_{i=1}^{H-1} \chi_i \frac{g(\varepsilon_i)}{p_i}$.

iii. The capital market clears: $\phi \cdot K = \int_0^1 s(j)dj$.

iv. The labor market clears: $\phi \cdot \sum_{h=1}^{H-1} N_h = z$.

Dynamic equilibrium

We are now ready to define the *dynamic equilibrium* of the economy.

DEFINITION 2. : Given an initial total capital amount owned by households, $\int_0^1 s_0(j)dj$, a *dynamic equilibrium* is a tuple $\left\{\mathbb{E}_t^T, \pi_t, d_t(j), c_t(j), s_{t+1}(j)\right\}$, $j \in [0,1]$, $t = 0,1,2,...$ such that:

1. Given $\int_0^1 s_t(j)dj$, $\mathbb{E}_t^T$ is a temporary equilibrium for all t.
2. Profits, π_t, are given by (12) evaluated at $K_t, \varepsilon_{1,t}, N_{1,t}, ..., \varepsilon_{H_t-1,t}, N_{H_t-1,t}$.
3. All profits are distributed to households: $\int_0^1 d_t(j)dj = \phi_t \cdot \pi_t$.
4. Household consumption and saving, $c_t(j)$ and $s_{t+1}(j)$, are determined by (7) and (8).

A Specification

Next we turn to a parameterized specification which is used to get some feeling about the mechanisms entailed in this model.

Functional forms

We set

$$F(K, \varepsilon N) = \theta K^{\alpha}(\varepsilon N)^{1-\alpha} \qquad \text{(Produnction)}$$

$$y(j) = \omega j^{\nu} \qquad \text{(Self-employed)}$$

$$g(\varepsilon) = \varepsilon^{\beta} \qquad \text{(Effort)}$$

$$P_h(\varepsilon m) = \frac{\mu_h \varepsilon m}{\mu_h \varepsilon m + 1}, \ \mu_{h+1} \leq \mu_h \qquad \text{(Probability)}$$

It is fairly easy to show (see Appendix A) that under specification (Effort) and (Probability) the monitoring probability is constant, given by

$$P_h(x) = 2 - \beta.$$

Accordingly, to obtain an interior solution we impose $1 < \beta < 2$. Furthermore, it turns out that given specification (Effort) and (Probability), imposing constant returns to scale on the production technology implies that effort at any level h is independent of H.[17]

Parameters

The following parameters are used to generate the economy's path presented below. Some of these parameters, such as time preference, product elasticity with respect to capital and the depreciation rate, are standard. Others are "calibrated". Specifically, given the aforementioned characteristics of hierarchical organizations, we aimed at obtaining steady-state levels of $H = 6$ (i.e.: five hierarchy levels below "management") and control spans of 6-12 workers. Table 1 reports the chosen parameter values:

Table 1: Parameter values

Preferences		Production			S.E output		Supervision	
η	β	θ	α	δ	ω	ν	μ_h	γ
$\frac{1}{1.03}$	1.8	2.7	0.3	0.08	2.75	4.81	$\frac{7.4}{2.6^h}$	1

The parameters ω and ν, which characterize the production function of the self-employed, are set to generate a steady-state fraction of "employees", z, of 0.9, as well as a ratio of 0.1 between the output of the self-employed relative to that of the "organized sector".[18] Specif-

[17] A similar result is obtained by Calvo and Wellisz (1979). It should be noted that this result implies that the hierarchical structure is economically inefficient (see also McAfee and McMillan 1995). We show elsewhere (Bental and Winer 2012) that with increasing returns to scale having a hierachy becomes efficient. We choose to report below the CRS case purely for numerical reasons.

[18] The fraction of employees corresponds to data presented by Bernanke and Gürkaynak (2001) Table10. The relative output of the self-employed is based on columns 2 and 3, Table B-10, in the Economic Report of the President, 2011.

ically,the structure of the firms and their output is fully determined by $\delta + r$. Since at steady-state $r = \frac{1}{\eta} - 1$, that value is given by the parameter choice. The ensuing expected labor income is equalized to the income of the marginal self-employed household at $z = 0.9$. Furthermore, the measure of firms is determined by the ratio between the pre-determined fraction of employees and per-firm employment. This sets the total output of the organized sector which, by assumption is ten times higher than the cumulated output of the upper 10% of the households choosing to be self-employed.

Results

The economy is started with 20% of the steady-state level of capital. The ensuing initial organizations are of the minimal scale (i.e., $H = 2$). The steady-state is reached after 31 periods, and the organizations have 5 hierarchy levels below "management" ($H = 6$).[19] Along the growth path household savings increase according to (8), thereby reducing the interest rate. Firms react to the lower interest rate by demanding more capital. Since labor complements capital, firms also hire more production workers and induce higher effort. Both changes require more supervisors, who are also induced to produce higher effort. At some stage it becomes worthwhile to add another supervisory level. Thus the firms' hierarchies become longer and control spans larger. Finally, since wages are positively related to effort levels, pay differentials along the hierarchical scale increase.

The increased demand for labor in the organized sector attracts workers away from self-employment, thereby reducing the size of the latter sector and leaving it with the better households. Therefore, productivity in the economy increases for two reasons: higher effort in the organized sector and better selection in the self-employed sector. Both reasons labor cause labor-income to increase along the growth path. Nevertheless, labor share is decreasing

[19] See Appendix B for a stylized description of the solution algorithm. We allow co-existence of firms of different sizes to equilibrate the economy. This happens in period 11, where two-level and three-level (below management) firms coexist, in period 20 where three- and four-level firms coexist ,and in periods 27 and 28, where four- and five-level firms coexist..

since due to capital deepening and improved average labor productivity output to grows even faster.

Performance

Figure 8 presents the development of (log) capital, output of the economy as a whole and output of each of the two sectors. The Figure shows that as the amount of capital in the economy increases, the contribution of the self-employed to total output is declining, reflecting the transition of workers into the organized sector (see also Figure 3 below). In fact, due to the transition from one to two level hierarchies in period 3 the economy is barely growing because "too many" workers become supervisors (see footnote 17).

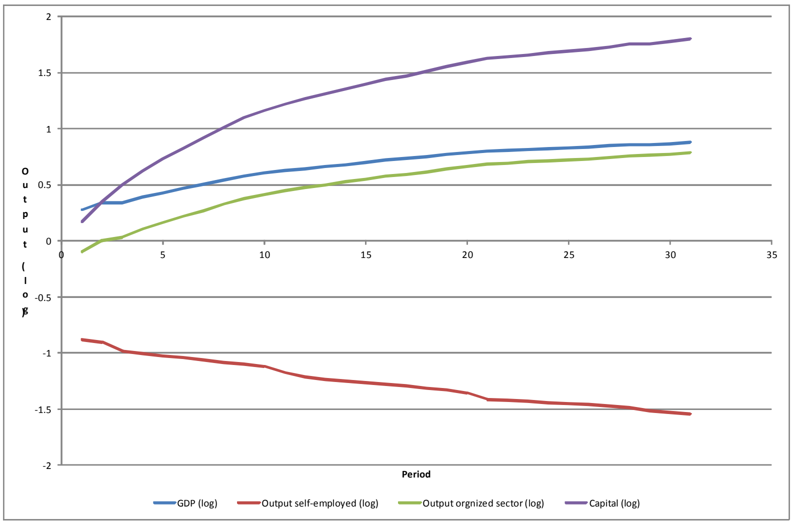

Figure 8: Capital and Output (log)

Figure 9 shows the development of the number of firms and the fraction of households which choose the "employee status". The periods in which additional hierarchical levels are added are clearly discernible. Such periods are characterized by upward "jumps" in the fraction of employees resulting from the sharp increase in wages, and sharp reduction in the number of firms which corresponds to their increased size.

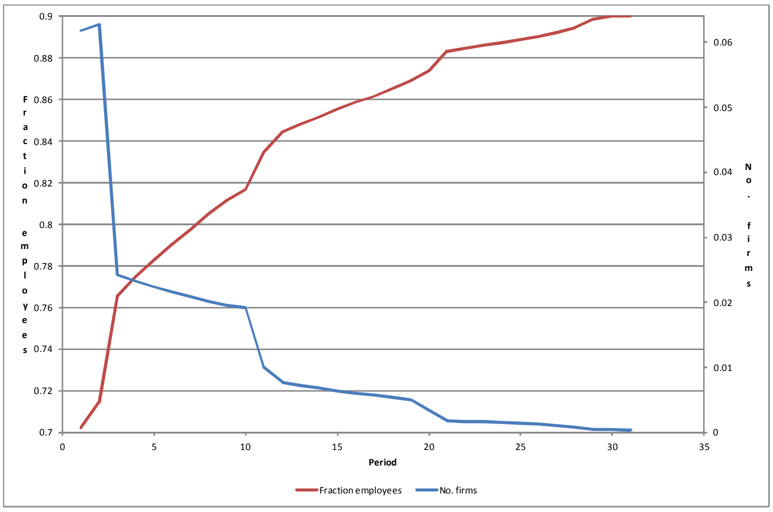

Figure 9: Fraction Employees and Number of Firms

Figures 10, 11 and 12 provide insight into the mechanisms involved. Figure 10 shows that the number of production workers in the firms is growing throughout (with "jumps" corresponding to the added supervisory levels). The number of supervisors at the respective top level (below "management") is constant .[20]. Thus it is obvious that their control span is growing, as shown by Figure 11. However, once any particular level is no longer at the top, the number of supervisors too is increasing. As an implication we obtain that as organizations grow, they also become "deeper"

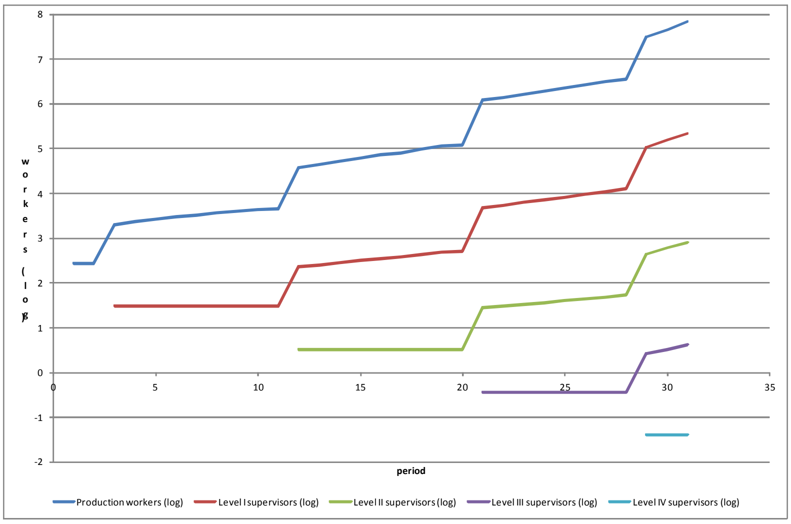

Figure 10: Number of Workers (log)

[20]This is a consequence of the CRS assumption.

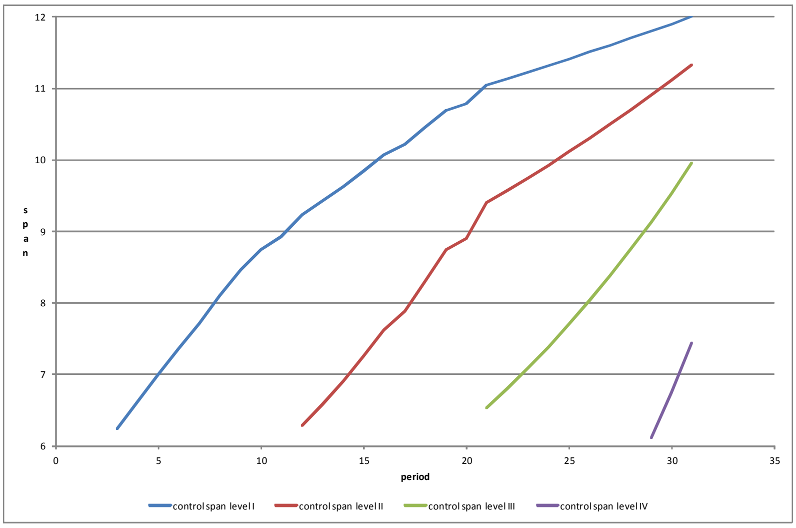

Figure 11: Control Spans

Finally, Figure 12 shows the effort levels. In order to induce ever higher effort on the production workers while reducing control over them, the supervisors too must be induced to exert more effort. Indeed, their effort must increase even faster than that of their subordinates. By analogous arguments we obtain the same results at all levels. Since wages are just proportional to the convex effort cost, the implication of Figures 12 is that wage gaps are increasing over time.[21] As a result, the compensation of workers, *relative* to that of their superiors, is declining. This predicition is in line with the pattern found by Caliendo, Monte and Rossi-Hansberg (2012).[22]

[21] Remember that $w_h = \frac{g(a_h)}{p_h}$ and that $p_h = 2 - \beta$ for all h.

[22] In contrast to the model presented above, the model of Caliendo, Monte and Rossi-Hansberg predicts that the introduction of a new hierarchical level causes *absolute* compensation of workers in lower levels to decline. However, the empirical study presented by these authors is considering *detrended* wages. In this sense the two models seem to be observationally equivalent.

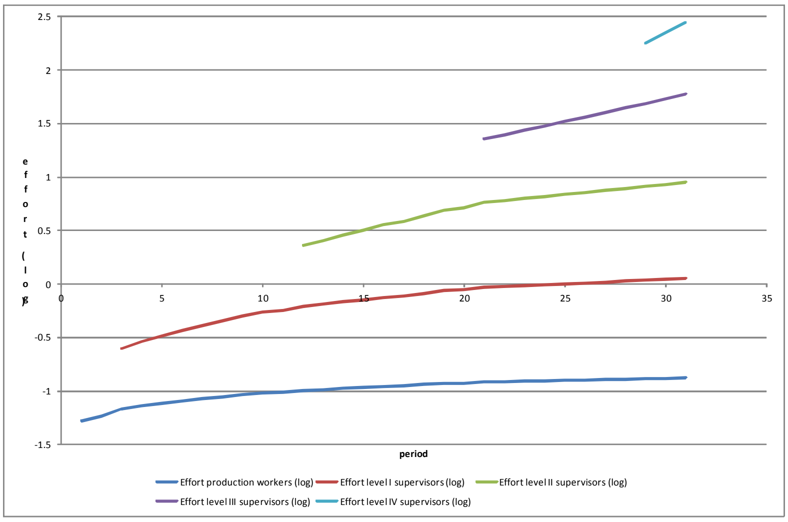

Figure 12: Effort (log)

Concluding Remarks

This paper uses a very rudimentary model oh hierarchical organizations. Specifically, the only role performed by upper levels of an organization is to monitor subordinates in order to prevent shirking. We have embedded these organizations in a simple growth environment and have shown that capital accumulation is sufficient to induce organizational changes. Specifically, more capital causes organizations to induce higher effort. This can be implemented either through higher wages or intensified monitoring. At some stage it becomes worthwhile for an organization to add a supervisory level. We have demonstrated that this setup can mimic some key features of the data. It corresponds to the historical growth trend of business organizations accompanied by the decline in their number, the improved productivity and growing wage gaps. It is also consistent with some key features found in the micro-data concerning the number of hierarchical levels and control spans.

It is quite clear that there are many questions which are left open. The simplicity of the model leaves out important roles performed by management. In particular, it does not consider information processing. Accordingly it is not well suited to address questions related to the impact of ICT on hierarchical structures. Thinking about transactions costs in conjunction with information processing is likely to provide new insights into organizational structures. Specifically, it may help explain some of the changes that seem to be taking place in organizational structures.

References

Bental, B. and D. Demougin (2006): "Incentive Contracts and Total Factor Productivity", *International Economic Review*, 47(3), 1021-1043.

Bental, B. and Z. Winer (2012): "Determinants of Hierarchical Structures", Department of Economics, University of Haifa, discussion paper (in preparation).

Bernanke, B. and R.S. Gürkaynak (2001): "Is Growth Exogenous? Taking Mankiw, Romer, and Weil Seriously", *NBER Macroeconomics Annual*,16(1), 11-57.

Bolton, P. and M. Dewatripont (1994): "The Firm as a Communication Network", *The Quarterly Journal of Economics*, 109(4), 809-839.

Caliendo, L. and E. Rossi-Hansberg (2012): "The Impact of Trade on Organization and Productivity", *The Quarterly Journal of Economics,* 27(3), 1363-1497.

Caliendo, L., F. Monte and E. Rossi-Hansberg (2012): "The Anatomy of French Production Hierarchies", NBER Working Paper No. 18259.

Calvo, G. and S. Wellisz (1979): "Hierarchy, Ability and Income Distribution", *Journal of Political Economy*, 87(5), 991-1010.

Colombo, M. G. and M. Delmastro (2008): *The Economics of Organizational Design: Theoretical Insights and Empirical Evidence*, Palgrave MacMillan.

Delmastro, M. (2002): "The Determinants of the Management Hierarchy: Evidence from Italian Plants", *International Journal of Industrial Organization,* 20, 119–137

Façanha, L.O. and M. Resende (2007): "Hierarchical Structure in Brazilian Industrial Firms: An Econometric Study", CESifo Working Paper No.1985.

Garicano, L. (2000): "Hierarchies and the Organization of Knowledge in Production", *Journal of Political Economy*, 108(5), 874–904.

McAfee, R. P. and J. McMillan (1995): "Organizational Diseconomies of Scale", *Journal of Economics and Management Strategy*, 4(3), 399-426.

Mookherjee, D. (2012): "Incentives and Hierarchy", *Handbook of Organizational Economics*, R.Gibbons and J.Roberts, eds., forthcoming. Princeton University Press.

Qian, Y. (1994): "Incentives and Loss of Control in an Optimal Hierarchy", *Review of Economic Studies*, 61(3), 527-544.

Radner, R. (1993): "The Organization of Decentralized Information Processing", *Econometrica*, 61(5), 1109-1146.

Rajan, R. and J. Wulf (2006): "The Flattening Firm: Evidence from Panel Data on the Changing Nature Corporate Hierarchies", *Review of Economics and Statistics*, 88(4), 759-783.

Williamson, O. (1967): "Hierarchical Control and Optimal Firm Size ", *Journal of Political Economy*, 75(2), 123-138.

Appendix A

The Firm's Optimization Problem

Omitting arguments wherever it does not lead to confusion, the firm's $FONC$s with respect to ε_1 and N_1 associated with (12) are given, respectively, by:

$$F_2 - \frac{g'(\varepsilon_1)}{P_1} = 0 \tag{A1}$$

$$F_2\varepsilon_1 - \frac{g(\varepsilon_1)}{P_1} - \frac{g(\varepsilon_1)}{(P_1)^2} P_1' \varepsilon_2 m_2 = 0 \tag{A2}$$

From here we easily obtain:

$$\frac{g'(\varepsilon_1)}{g(\varepsilon_1)}\varepsilon_1 - \left[1 + \frac{P_1'}{P_1}\varepsilon_2 m_2\right] = 0 \tag{A3}$$

For level $h > 1$ optimization with respect to ε_h and N_h requires:

$$-\frac{g'(\varepsilon_h)}{P_h} + \frac{g(\varepsilon_{h-1})}{(P_{h-1})^2} P_{h-1}' = 0 \tag{A4}$$

$$-\frac{g(\varepsilon_h)}{P_h} + \frac{g(\varepsilon_{h-1})}{(P_{h-1})^2} P_{h-1}' \varepsilon_h - \frac{g(\varepsilon_h)}{(P_h)^2} P_h' \varepsilon_{h+1} m_{h+1} = 0 \tag{A5}$$

By simple manipulation we obtain that here too:

$$\frac{g'(\varepsilon_h)}{g(\varepsilon_h)}\varepsilon_h - \left[1 + \frac{P_h'}{P_h}\varepsilon_{h+1} m_{h+1}\right] = 0 \tag{A6}$$

Using the specification of the monitoring probability (Probability), it is easy to show that:

$$\frac{P_h'}{P_h}\varepsilon_{h+1} m_{h+1} = 1 - P_h \tag{A7}$$

Conclusion 1: With effort cost (Effort), the supervision probability is constant and identical at all levels.

Proof: Conditions (A3) or (A6) simplify to:

$$\beta - [2 - P_h] = 0,$$

implying, of course:

$$P_h = 2 - \beta, \ \forall \ h. \tag{A8}$$

Accordingly, to obtain an internal solution we must require $1 < \beta < 2$.

Consider a production function (9) which is characterized by constant returns to scale. Under this condition, substituting the $FONC$ with respect to K into F_2 yields a constant. We then obtain:

Conclusion 2: Due to (A8), the effort level at the bottom, ε_1, is independent of the number of workers (see A1).

Conclusion 3: The effort level at any level $h < H$ is independent of the value of H and is determined recursively from bottom to top. Specifically, ε_2 is determined by (A5), using conclusions 1 and (A8). Then ε_3 is determined by ε_2, etc.

Conclusion 4: The inverse control spans are determined recursively from top to bottom. Starting at level $H - 1$, from (Probability) and we compute $\mu_{H-1}\varepsilon_H m_H = \frac{2-\beta}{\beta-1}$. By the definition of m_H and given that $\varepsilon_H N_H$ is given by Assumption 1we obtain $N_{H-1} = \mu_{H-1}\gamma\frac{\beta-1}{2-\beta}$. Since ε_{H-1} has already been determined, we now know the value of $\mu_{H-2}\varepsilon_{H-1}N_{H-1}$ and the process can be repeated. In general we have:

$$N_h = \mu_h \varepsilon_{h+1}\frac{\beta - 1}{2 - \beta}N_{h+1}, \ h = 1, ..., H - 1 \tag{A9}$$

Conclusion 5: The inverse control spans m_{h+1}, $h = 1, ..., H - 1$, are independent of γ.

Corollary: The optimal number of hierarchical levels is independent of γ.

The only impact of changes in γ is to proportionally change the number of workers at all levels. Therefore profits are proportional to γ.

Remark : If the production technology is characterized by increasing returns to scale, conclusions 2-5 are no longer valid. In particular, effort at all levels is positively related to the (endogenously determined) number of workers. Furthermore, "management efficiency", γ, affects the optimal number of hierarchical levels.

Appendix B

Flow-chart of the solution algorithm